Rules for Differentiation Calculus of a Single Variable X. Du

- Derivatives can be evaluated as a sum of its individual terms.
  - Let f and g be two functions of x, then $\frac{d}{dx}[f(x)+g(x)] = f'(x)+g'(x)$
- $\frac{d}{dx}(cf(x)) = cf'(x)$
- **Power rule**: $\frac{d}{dx}(x^n) = nx^{n-1}$ By extension: $\frac{d}{dx}(cx^n) = cx^{n-1}$
- $\frac{d}{dx}\sin x = \cos x$ $\frac{d}{dx}\cos x = -\sin x$
- **Product rule**: $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$
- **Quotient rule**: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x)+g'(x)f(x)}{[g(x)]^2}$ for $g(x) \neq 0$
- $\frac{d}{dx}\tan x = \sec^2 x$ $\frac{d}{dx}\cot x = -\csc^2 x$
- $\frac{d}{dx}\sec x = \sec x \tan x$ $\frac{d}{dx}\csc x = -\csc x \cot x$
- $\frac{d}{dx}e^x = e^x$ $\frac{d}{dx}\ln x = \frac{1}{x}$
- **Chain rule**: $\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$
  - In other words: If y varies with u and u varies with x, then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
  - Think about this intuitively: if y is three times as fast as u and u is twice as fast as x, then y must be six times as fast as x.

Further notes:

- **Leibniz's Rule**: Given that u and v are n times differentiable functions,

  $(uv)^{(n)} = \sum_{k=0}^{n} \left({}^nC_k\right) u^{(k)} v^{(n-k)}$ Note: this is just the generalization of the product rule.
- The derivatives of the sine and cosine functions can be proven with the limit definition of the derivative with trigonometric summation properties using $\lim_{x\to 0}\frac{\sin x}{x} = 1$ and $\lim_{x\to 0}\frac{1-\cos x}{x} = 0$.
- The derivatives of the other trigonometric functions can be derived from the derivative of the sine and cosine functions using product and quotient rule.
- The derivatives of exponential functions can be proven with the definition of e.
- Product rule, quotient rule, and chain rule can all be proven with the limit definition of the derivative.